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DIFFRACTION BY A CIRCULAR ISLAND OF LONG WAVES PRODUCED BY A RIPARIAN SOURCE

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The diffraction of long cylindrical waves by a circular island situated in a rotating tank is considered. It is shown that, when the wavelength is small in comparison with the island radius, a resonanace capture of waves by the island takes place. Unlike in the author's paper [1] which analyzed the diffraction of monochromatic plane waves by a circular island in a rotating tank of constant depth, here the diffraction of cylindrical waves produced by a source at the island boundary is considered. As in [1], the wavelength is assumed to be considerable in comparison with the depth of the tank, but small relative to the island radius. A solution in the form of a conventional slowly convergent Fourier series is first derived, and then transformed by Watson's method into a fast convergent series, which makes it possible to determine the pattern of wave diffraction, at least along the island periphery. Many details of the derivation of solution have been omitted here. One of these details can be found in [1, 2], while others may be obtained by small alterations in the calculations presented in those papers.

1. Statement of problem. Fourier series for the elevation of fluid. A horizontally unbounded tank filled with a heavy perfect fluid rotates at angular velocity ω in a counterclockwise direction about a vertical axis. Depth of the tank is throughout uniform and equal h. The tank contains a source generating cylindrical waves at frequency $\sigma > 2\omega$ and an "island" – a rigid circular cylinder of radius a – with its axis parallel to the tank axis of rotation. Let us examine the motion of fluid along the island periphery, and begin by assuming that the source of waves lies outside the island periphery at a distance b from the island center. We then consider the case in which the source is right up the island periphery, i.e., we effect the transition to limit $b \to a$.

We introduce a polar system of coordinates ρ, θ in the plane in which the surface of fluid would lie in the absence of tank rotation and wave sources. We locate the pole at the point of intersection of this plane with the cylinder axis, i.e., at the island center, and draw the polar axis through the source. We denote the elevation of fluid by $\zeta(\rho, \theta)\, e^{i\sigma t}$ and by $\zeta_1(\rho, \theta) e^{i\sigma t}$ that of its part which is due exclusively to the effect of the source without allowance for the reflection of waves by the island.

Function $\zeta_1(\rho, \theta)$ is of the form

$$\zeta_1(\rho, \theta) = AH_0^{(2)}(x), \quad x = \varkappa(\rho^2 + b^2 - 2\rho b \cos\theta)^{1/2}, \quad \varkappa = \left(\frac{\sigma^2 - 4\omega^2}{gh}\right)^{1/2}$$

where A is the amplitude of waves produced by the source, $H_0^{(2)}(x)$ is the Hankel function of the second kind and order zero, and g is the acceleration of gravity.

Function $\zeta(\rho, \theta)$ must be a solution of equation

$$\frac{\partial^2\zeta}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\zeta}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\zeta}{\partial\theta^2} + \varkappa^2\zeta = 0 \quad \text{for } \rho > a \tag{1.1}$$

satisfy at the cylinder rigid wall the condition

$$\rho\frac{\partial\zeta}{\partial\rho} - \frac{2i\omega}{\sigma}\frac{\partial\zeta}{\partial\theta} = 0 \quad \text{for } \rho = a \tag{1.2}$$

and at point $(b, 0)$ have the same singularity as ζ_1. Furthermore, the conditions of Sommerfeld radiation must be satisfied by the remainder

$$\zeta_2 = \zeta - \zeta_1$$

Let us rewrite $\zeta_1(\rho, \theta)$ as follows:

$$\zeta_1(\rho, \theta) = A\sum_{n=-\infty}^{\infty} e^{in\theta}\begin{cases} J_n(\varkappa\rho)\, H_n^{(2)}(\varkappa b), & \rho < b \\ J_n(\varkappa b)\, H_n^{(2)}(\varkappa\rho), & \rho > b\end{cases}$$

where $J_n(x)$ is a Bessel function, and seek $\zeta_2(\rho, \theta)$ in the form

$$\zeta_2(\rho, \theta) = A\sum_{n=-\infty}^{\infty} B_n H_n^{(2)}(\varkappa\rho)\, e^{in\theta}$$

Each term of this series satisfies Eq. (1.1) and the conditions of radiation. From condition (1.2) we find that

$$B_n = -\frac{H_n^{(2)}(\varkappa b)}{\psi(n)}\left[\frac{2\omega}{\sigma} nJ_n(\varkappa a) + \varkappa a\frac{\partial J_n(\varkappa a)}{\partial\varkappa a}\right]$$

$$\psi(\nu) = \frac{2\omega}{\sigma}\nu H_\nu^{(2)}(\varkappa a) + \varkappa a\frac{\partial H_\nu^{(2)}(\varkappa a)}{\partial\varkappa a}$$

Setting $\zeta_1(\rho, \theta)$ and adding $\zeta_2(\rho, \theta)$ to $\rho = a$, we obtain

$$\zeta(a, \theta) = -\frac{2iA}{\pi}\sum_{n=-\infty}^{\infty}\frac{H_n^{(2)}(\varkappa b)}{\psi(n)}e^{in\theta} \tag{1.3}$$

Series (1.3) is obviously absolutely convergent, when $b > a$, and the convergence in any interval $b_1 \leqslant b \leqslant b_2$, where $b_1 > a$ is uniform.

To investigate the case of a riparian source it is necessary to make b tend to a and find the limit of the right-hand part of (1.3). One can hardly expect to find this limit by passing to limit for each term of series (1.3), since for large $|n|$ and b equal a these terms behave as $e^{in\theta}/n$, hence it is not even clear whether this series is convergent. Because of this we leave for the time being the case of the riparian source, and shall consider b to be positively greater than a.

The convergence rate of series (1.3) depends on parameters $\varkappa a$ and $2\omega/\sigma$. When $\varkappa a$ is small and the ratio $2\omega/\sigma$ not too close to unity, the series rapidly converges. If, on the other hand, $\varkappa a$ is considerable, the rate of convergence is slow, and in summating the series it is necessary to take more than $2\varkappa a$ terms, which is the case in our problem. Indeed, the ratio of the ratio of the island radius to the length of waves produced by the source is, by assumption, considerable and $\varkappa a$ is equal to this ratio multiplied by 2π.

We use the Watson method [3, 4] for investigating the slowly convergent series (1.3) and, first, consider the distribution of zeros of function $\psi(\nu)$ in plane ν.

2. On the zeros of function $\psi(\nu)$. Since $\psi(\nu)$ is an entire function, it has an infinite number of zeros. That all of these are complex is shown in [1], where formulas derived on the assumption of constant nonzero parameter $2\omega/\sigma$ and $\varkappa a \gg 1$, are presented. Here we consider the case of constant $\varkappa a \gg 1$, and of parameter $2\omega/\sigma$ varying from zero to unity. As in [1], the whole of plane is divided into a number of regions, each of which is considered separately.

In "remote" regions of plane ν, such that

$$|\nu| \gg \max\{\varkappa a,\ \ln(1 - 2\omega/\sigma)\}$$

the zeros of function $\psi(\nu)$ are defined, as in [1], by formulas

$$\nu_n^{\pm} = \frac{(n \pm {}^1\!/_4)\,\pi}{\ln \dfrac{2\pi(n \pm {}^1\!/_4)}{\varkappa a e}} \exp\left\{\pm\frac{1}{2}\pi i\left[1 \pm \frac{1}{\ln \dfrac{2\pi(n \pm {}^1\!/_4)}{\varkappa a}}\right]\right\} \times \tag{2.1}$$

$$\times\left[1 + O\left(\frac{\ln\ln n}{\ln n}\right)\right], \qquad n = N, N+1, \ldots \quad (N \gg 1)$$

Among the remaining zeros of function $\psi(\nu)$ of considerable interest are those which have least absolute values of their imaginary parts. Below we deal only with zeros of that kind.

In the "transition" region of the right-hand half-plane defined by formulas

$$\nu - 2^{-1/3} e^{2/3 \pi i} z \nu^{1/3} = \varkappa a, \qquad \nu = \varkappa a - 2^{-1/3} e^{-1/3 \pi i} z (\varkappa a)^{1/3} + o(1) \tag{2.2}$$

where z is a complex number satisfying the only one condition $|z| \ll (\varkappa a)^{2/3}$, and the Hankel functions $H_\nu^{(2)}(\varkappa a)$ are expressed in terms of Airy function $\mathrm{Ai}(z)$. The determination of zeros of function $\psi(\nu)$ is now reduced to determining z from the equation

$$2^{1/3} e^{1/3 \pi i}\, \mathrm{Ai}'(z) + \frac{2\omega}{\sigma}(\varkappa a)^{1/3}\, \mathrm{Ai}(z) + O\left[(\varkappa a)^{-2/3}\right] = 0 \tag{2.3}$$

Let us narrow the region of variation of parameter $2\omega/\sigma$ and assume that one of the inequalities

$$0 \leqslant 2\omega/\sigma \ll (\varkappa a)^{-1/3} \tag{2.4}$$

$$(\varkappa a)^{-1/3} \ll 2\omega / \sigma < 1 \tag{2.5}$$

holds.

If the inequality (2.4) is valid, the solution of Eq. (2.3), for which ν derived by formula (2.2) have least absolute values of their imaginary parts, can be sought in the form of expansions

$$z_{n-1} = b_n + c_n\varepsilon + d_n\varepsilon^2 + \ldots, \quad \varepsilon = 2\omega / \sigma(\varkappa a)^{1/3}, \quad n = 1, 2, \ldots, N \tag{2.6}$$

where b_n are zeros of function $\mathrm{Ai}'(z)$ numbered in the ascending order of their absolute values and c_n, d_n etc. are unknown coefficients.

The substitution of expansions (2.6) into (2.3) yields $c_n = 2^{-1/3}e^{2/3\pi i}b^{-1}$ hence

$$\begin{gathered}\nu_{n-1}^{-} = p_n - iq_n, \quad n = 1, 2, \ldots, N \\ p_n = [\varkappa a + 2^{-4/3}(\varkappa a)^{1/3}(-b_n)][1 + o(1)] \\ q_n = 2^{-4/3}3^{1/2}(\varkappa a)^{1/3}(-b_n)[1 + o(1)]\end{gathered} \tag{2.7}$$

The zeros in the transition region of the left-hand half-plane with condition (2.4) satisfied are defined by formula

$$\nu_n^{+} = -p_n + iq_n, \qquad n = 1, 2, \ldots, N \tag{2.8}$$

It will be seen from (2.7) and (2.8) that in the "transition" regions the imaginary parts of zeros of function $\psi(\nu)$ are of the order of $(\varkappa a)^{1/3}$. In other regions of plane ν with condition (2.4) satisfied there are no zeros with smaller in absolute value imaginary parts.

If inequalities (2.5) are satisfied, then a procedure similar to that described above yields formulas

$$\begin{gathered}\nu_n^{\pm} = \mp r_n \pm is_n, \qquad n = 1, 2, \ldots, N \\ r_n = [\varkappa a + 2^{-4/3}(\varkappa a)^{1/3}(-a_n)][1 + o(1)] \\ s_n = 2^{-4/3}3^{1/2}(\varkappa a)^{1/3}(-a_n)[1 + o(1)]\end{gathered} \tag{2.9}$$

where a_n denote zeros of function $\mathrm{Ai}(z)$ numbered in the ascending order of their absolute values.

When inequality (2.5) is satisfied, function $\psi(\nu)$ has one more zero, namely ν_0^{-} whose imaginary part is small in comparison with the imaginary parts of zeros $\nu_n^{\pm}$. This zero lies in that part of plane ν in which the asymptotic Debye formulas apply to Hankel functions. If we assume $\nu = \varkappa a \operatorname{ch}\gamma$, then in that region $\operatorname{Re}\gamma < 0$ and $0 \leqslant \operatorname{Im}\gamma < 1/2\pi$, and for considerable values of the argument the Hankel functions are expressed in terms of exponents

$$e^{\pm f(\gamma, \varkappa a)}, \qquad f(\gamma, \varkappa a) = \varkappa a(\operatorname{sh}\gamma - \gamma\operatorname{ch}\gamma)$$

Only exponen $e^{f(\gamma, \varkappa a)}$ was considered in [1], where by means of series expansion in powers of parameter $(\varkappa a)^{-1}$ the real part of the zero ν_0^{-} was found to be

$$\operatorname{Re}\nu_0^{-} = \varkappa a\left[1 - \left(\frac{2\omega}{\sigma}\right)^2\right]^{-1/2} + \frac{\sigma}{4\omega} + O\left(\frac{1}{\varkappa a}\right) \tag{2.10}$$

The imaginary part $\operatorname{Im}\nu_0^{-}$ of this zero proved to be infinitely small, of an order of smallness higher than that of $(\varkappa a)^{-1}$ in any positive integral power. It was not determined in [1]. If, however, $\operatorname{Im}\nu_0^{-}$ is sought as an expansion in the following powers:

$$e^{-2f(\gamma_0,\ \varkappa a)}, \qquad \gamma_0 = \operatorname{arch}\frac{\operatorname{Re}\nu_0^-}{\varkappa a}$$

and in the Debye expansions both exponents are retained, then for $\operatorname{Im}\nu_0^-$ we obtain the expression

$$\operatorname{Im}\nu_0^- \approx -2\varkappa a\left(\frac{2\omega}{\sigma}\right)^2\left[1-\left(\frac{2\omega}{\sigma}\right)^2\right]^{-3/2} e^{-2f(\gamma_0,\ \varkappa a)} \tag{2.11}$$

$$f(\gamma_0, \varkappa a) = \varkappa a\left[1-\left(\frac{2\omega}{\sigma}\right)^2\right]^{-1/2}\left(\operatorname{arth}\frac{2\omega}{\sigma}-\frac{2\omega}{\sigma}\right)+\frac{\sigma}{4\omega}\operatorname{arth}\frac{2\omega}{\sigma}+O\left(\frac{1}{\varkappa a}\right)$$

Expression (2.11) can only be obtained, if the supplementary condition

$$1 - 2\omega / \sigma \gg (\varkappa a)^{-1} \tag{2.12}$$

is satisfied, but unfortunately less exactly than for the remaining zeros $\nu_n\pm$.

The inequalities (2.4) and (2.5) are not valid for those values of parameter $2\omega / \sigma$ which lie in the vicinity of $2\omega / \sigma = (\varkappa a)^{-1/3}$ and for which it would be hardly possible to determine the zeros of function $\psi(\nu)$ without resorting to methods of numerical analysis.

3. Rapidly converging series for wave elevation. Let us revert to formula (1.3) and, following Watson's method, substitute in it an integral for its right-hand side series. By analogy to [1] we obtain

$$\zeta(a, \theta) = -\frac{A}{\pi}\int\limits_{L_1+L_2}\frac{H_\nu^{(2)}(\varkappa b)}{\psi(\nu)}\frac{e^{i\nu(\theta-\pi)}}{\sin\nu\pi}d\nu \tag{3.1}$$

where $L_{1,2}$ is a straight line connecting in the complex plane ν points $(\pm\infty, \pm ip)$ and $(\mp\infty, \pm ip)$, where p is a positive number sufficiently small for any zeros of function $\psi(\nu)$ to be absent between L_1 and L_2.

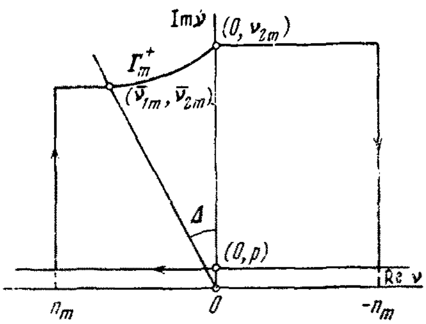

Fig. 1.

Let us calculate the integral (3.1) by the residues in the zeros of function $\psi(\nu)$. To do this we close the straight lines L_1 and L_2 by sequences of curves C_m^+ and C_m^-, respectively. As the C_m^+ curve in the sector $\frac{1}{2}\pi \leqslant \arg\nu \leqslant \frac{1}{2}\pi + \Delta$, where $0 < \Delta < \frac{1}{4}\pi$, we take arcs of curves Γ_m^+ defined, as in [1], by the equation

$$\operatorname{Im}\left[\nu\left(\ln\nu - \ln\frac{\varkappa ae}{2} - \pi i\right)\right] = m\pi, \quad m = M, M+1, \ldots \quad (M \gg 1)$$

The remaining parts of curves are made up of straight line segements, as shown in Fig. 1, where $n_m = E(\bar{\nu}_{1m}) - 3/2$. For C_m^- we take curves symmetric to C_m^+ about the coordinate origin.

We denote the absolute value of the integrand of (3.1) by $F(\nu)$, and by $\Pi_m^\pm$ the set of straight-line segments reaching $C_m^\pm$. Along $\Gamma_m^\pm$ and $\Pi_m^\pm$ we have for $F(\nu)$ the following estimates:

$$F(\nu) \leqslant C \begin{cases} \exp[-\theta \operatorname{Im}\nu - \operatorname{Re}\nu \ln(b/a)], & \nu \in \Gamma_m^+ \\ \exp[(2\pi - \theta)\operatorname{Im}\nu + \operatorname{Re}\nu \ln(b/a)], & \nu \in \Gamma_m^- \\ \exp[-\theta \operatorname{Im}\nu - |\operatorname{Re}\nu| \ln(b/a)], & \nu \in \Pi_m^+ \\ \exp[(2\pi - \theta)\operatorname{Im}\nu - |\operatorname{Re}\nu| \ln(b/a)], & \nu \in \Pi_m^- \end{cases} \tag{3.2}$$

(Here and in the following C denotes positive constants).
These estimates were obtained by the method described in detail in [2] with inequalities

$$|\sin \nu\pi|^{-1} \leqslant \begin{cases} 1, & |\operatorname{Im}\nu| \leqslant {}^1/_2, \quad \nu \in C_m^{\pm} \\ Ce^{-\pi|\operatorname{Im}\nu|}, & |\operatorname{Im}\nu| > {}^1/_2 \end{cases}$$

taken into account.

The estimates for the lengths $l(\Gamma_m)$ and $l(\Pi_m)$ (see [1]) are:

$$l(\Gamma_m) \leqslant \sqrt{2}\,|\bar{\nu}_{1m}|, \qquad l(\Pi_m) \leqslant C\,|\bar{\bar{\nu}}_{2m}| \tag{3.3}$$

Let us assume that angle θ is contained in the interval

$$\varepsilon \leqslant \theta \leqslant 2\pi - \varepsilon, \qquad 0 < \varepsilon < \pi \tag{3.4}$$

From the inequalities (3.2) and (3.3) then follows that sequences of integrals of the kind of (3.1) tend to zero along curves $C_m^{\pm}$, when m is indefinitely increased. This means that the integral (3.1) along lines L_1 and L_2 can be reduced to the sum of residues in the zeros $\nu_m^{\pm}$ of function $\psi(\nu)$ and we may consequently write

$$\zeta(a, \theta) = 2iA \sum_{n=0}^{\infty} \left[H_\nu^{(2)}(\varkappa b)\, e^{i\nu(\theta-\pi)} \left(\frac{\partial\psi}{\partial\nu}\sin\nu\pi\right)^{-1}\right]_{\nu=\nu_n^{\pm}} \tag{3.5}$$

The terms of this series whose order exceeds a certain sufficiently high natural number N are defined for $\nu = \nu_n^{\pm}$ to within infinitely small magnitudes by formula

$$\zeta_n^+ \sim -i\left[\left(1 - \frac{2\omega}{\sigma}\right)\nu_n^+ \ln \nu_n^+\right]^{-1} \exp\left(i\nu_n^+\theta - \nu_n^+ \ln\frac{b}{a}\right) \tag{3.6}$$

For $\nu = \nu_n^-$ the terms of this series are defined by a formula derived from (3.6) by the substitution of ν_n^- for ν_n^+, $2\pi - \theta$ for θ and $-2\omega/\sigma$ for $2\omega/\sigma$. If $a \leqslant b \leqslant b_1$ is assumed and formulas (2.1) taken into account, it becomes possible to establish that the absolute values of the Nth order terms of series (3.5) are smaller than the corresponding terms of series

$$C \sum_{n=N}^{\infty} \frac{1}{n} \left\{\exp\left[-\frac{n\pi}{\ln n}(\theta - \delta)\right] + \exp\left[-\frac{n\pi}{\ln n}(2\pi - \theta + \delta)\right]\right\} \tag{3.7}$$

$$\delta \leqslant \min(\theta, 2\pi - \theta)$$

For angles θ comprised in the interval (3.4) series (3.7) is convergent. Hence for these angles series (3.5) is also convergent and, with respect to b within the segment $[a, b_1]$ it is uniformly convergent.

In the case of a riparian source, which henceforth will be dealt with, it is possible to pass in the series (3.5) to the limit $b \to a$ term by term, and set $b = a$. The basic contribution to the sum of series thus derived is provided by the terms which correspond to the $\nu_n^{\pm}$ zeros with the least in absolute value imaginary parts.

If parameter $2\omega/\sigma$ satisfies the inequality (2.4), the zeros $\nu_n^{\pm}$ which have the above properties are defined by formulas (2.7) and (2.8), and the approximate expression for $\zeta\ (a,\ \theta)$ is of the form

$$\zeta(a,\theta) \approx \frac{2^{4/3}e^{1/3\pi i}A}{(\varkappa a)^{1/3}} \sum_{n=1}^{N} \frac{1}{-b_n} \left[e^{-\theta(q_n+ip_n)} + e^{-(2\pi-\theta)(q_n+ip_n)}\right] \tag{3.8}$$

If, however, parameter $2\omega\ /\ \sigma$ satisfies inequalities (2.5) and (2.12), the zeros $\nu_n^{\pm}$ with the least in absolute value imaginary parts are determined by formulas (2.9) - (2.11) while the approximate expression for $\zeta\ (a, \theta)$ is written as

$$\zeta(a,\theta) \approx -\frac{4\omega}{\sigma}\frac{iA}{\sin\nu_0^-\pi}\left[1-\left(\frac{2\omega}{\sigma}\right)^2\right]^{-1} e^{i\nu_0^-(\theta-\pi)} - \tag{3.9}$$

$$-\frac{4A}{\varkappa a}\left(\frac{\sigma}{2\omega}\right)^2 \sum_{n=1}^{N}\left[e^{-\theta(s_n+ir_n)} + e^{-(2\pi-\theta)(s_n+ir_n)}\right]$$

Real parts of the power of exponents in (3.8) and (3.9) are proportional to q_n and s_n defined by formulas (2.7) and (2.9). Parameters q_n and s_n are also propertional to $(\varkappa a)^{1/3}$ and to the positive numbers $-b_n$ and $-a_n$ For $n < 5$ these numbers increase approximately by unity with increasing n by unity. For $n \gg 5$ the numbers $-a_n$ and $-b_n$ increase with increasing n approximately as $n^{2/3}$. Consequently for considerable $\varkappa a$ the values q_n and s_n rapidly increase with increasing n. If it is further assumed that the inequalities

$$\varepsilon \leqslant \theta \leqslant 2\pi - \varepsilon, \qquad (\varkappa a)^{-1/3} \ll \varepsilon < \pi \tag{3.10}$$

are satisfied, then the absolute values of terms in the summations (3.8) and (3.9) are rapidly decreasing with increasing n.

Thus the slowly convergent Fourier series at considerable $\varkappa a$ has been transformed into the rapidly convergent series (3.8) and (3.9).

4. Physical interpretation of results. Formulas (3.8) and (3.9) make it possible to obtain a clear picture of the wave motions of fluid along the island periphery and then trace how these motions are affected by the rotation of the tank.

Formula (3.8) applies to the case of slow or altogether absent rotation of the tank. The exponents of this formula define conventional diffracted waves which appear to radiate from the point $\theta = 0$ and flow around the island in clock- and counterclockwise directions. The amplitude of these waves rapidly diminishes with increasing distance from point $\theta = 0$. The diffraction pattern closely resembles that of diffraction of short electromagnetic waves by a dipole on the surface of a perfectly conducting cylinder.

For faster tank rotation formula (3.9) is applicable. It will be seen from it that alongside the conventional rapidly attenuated diffraction waves, a separate wave, defined by the first term in the right-hand side of formula (3.9), circulates around the island in a clockwise direction, virtually without attenuation, since according to (2.10) and (2.11) the inequalities $\mathrm{Re}\,\nu_0^- > 0$ and $|\ \mathrm{Im}\ \nu_0^-\ | \ll 1$. The amplitude of this particular wave, owing to the presence of $\sin\nu_0^-\pi$ in the denominator of formula (3.9), may prove to be very great, if the relationship $\mathrm{Re}\,\nu_0^- = N$, where N is a natural number exceeding $\varkappa a$ by at least a factor of $(\varkappa a)^{1/3}$.

Thus, as in the case of plane waves, a resonance capture by the island of waves emanating from a source takes place.

In off-resonance modes the amplitude of this particular wave increases with increaseing $2\omega/\sigma$ in the case of diffraction of waves generated by a source, and decreases in that of plane waves.

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NONLINEAR ACOUSTICS OF CHEMICALLY ACTIVE MEDIA

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Nonlinear propagation of disturbances is examined in reacting mixtures where the change of composition is determined by the course of a single chemical reaction. Depending on the relationship between macroscopic time and relaxation time, we distinguish two basic types of processes: quasi-frozen and quasi-equilibrium. Media are examined also, in which the frozen and equilibrium speeds of sound are nearly equal in magnitude. Solutions are constructed for asymptotic equations which describe the flow parameters behind shock fronts and in expansion waves. A mathematical analogy is formulated for the effect of rates of chemical reactions, the effect of "longitudinal viscosity", and the effect of thermal conductivity on the structure of the perturbed field.

1. Initial equations. It will be assumed that in the flow of chemically active gas mixture only one reaction takes place. The change in the composition of the mixture is then characterized by a single parameter q which is called completeness of reaction.

The equations of motion of the mixture are taken in the form [1]

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + (\nu - 1)\frac{\rho v}{r} = 0, \qquad \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$Q\left(\frac{\partial q}{\partial t} + v\frac{\partial q}{\partial r}\right) + T\left(\frac{\partial s}{\partial t} + v\frac{\partial s}{\partial r}\right) = 0, \qquad \frac{\partial q}{\partial t} + v\frac{\partial q}{\partial r} = q^{\cdot} \tag{1.1}$$

Here t is the time, r is the distance from the plane, axis or center of symmetry, v is the velocity, ρ is the density, p is the pressure, s is the specific entropy, T is the temperature, q and Q are the rate and affinity of chemical reaction. The parameter $\nu = 1, 2, 3$ for flows with a plane, axis or center of symmetry, respectively.

In order to close the system it is necessary to introduce three additional equations which connect thermodynamic functions q, ρ, p, s, Q and According to the Gibbs relationship the increase in specific internal energy e is